Positioning Accuracy Improvement Effect Based on Multiple Observations of Doppler shift for User Position Detection System Using Unmanned Aerial Vehicles

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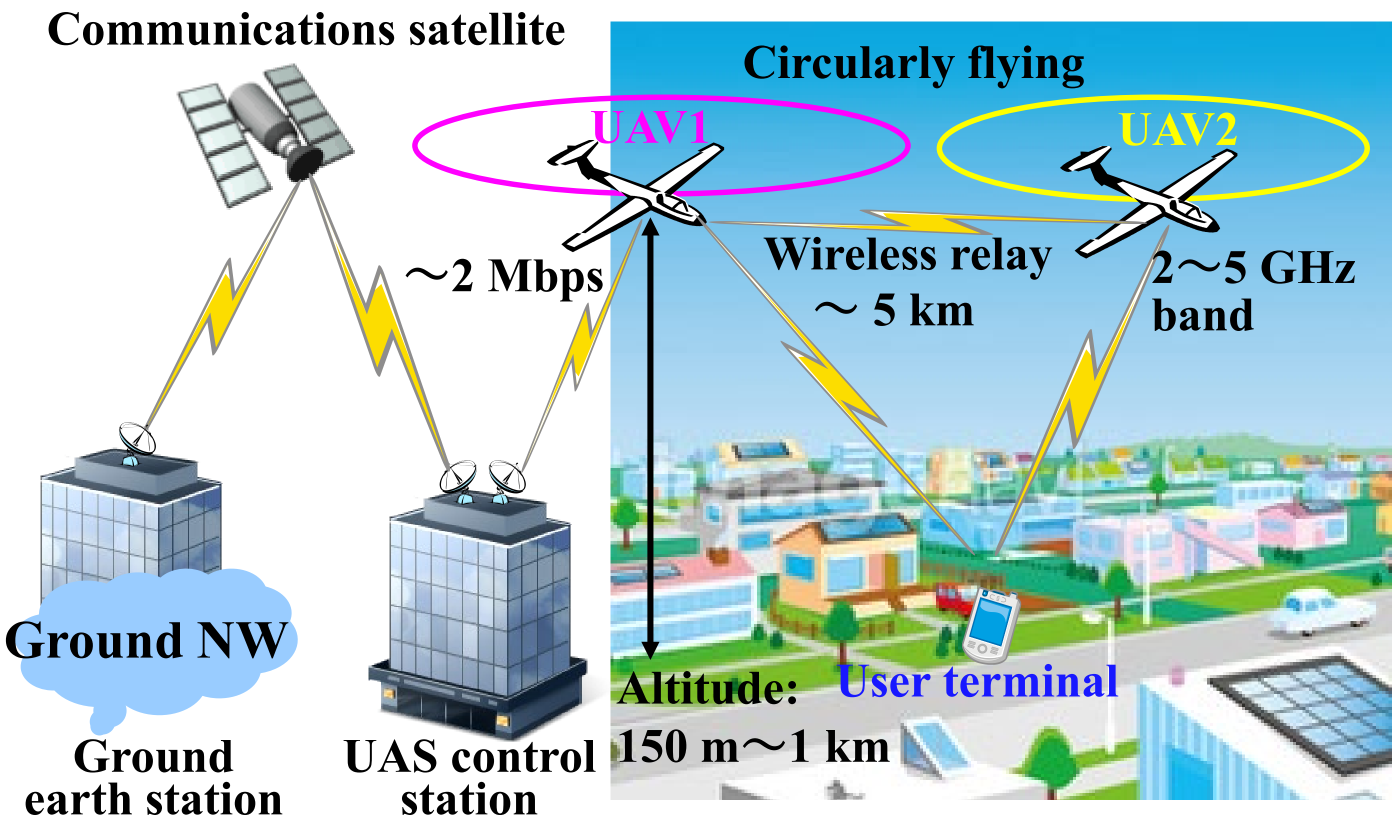
Background



Example system configuration model for UAS

- Unmanned aircraft system (UAS) provides temporal communication network using unmanned aerial vehicles (UAVs) for emergency and rescue service during disasters.
- Doppler shift is occurred in the carrier frequency of the transmission signals.
- By observing multiple Doppler shifts for UAVs, it is possible to detect the user position.

Proposed methods so far

- **User position detection method using multiple UAVs by observing multiple Doppler shifts at once or single UAV by observing Doppler shift at different times**
  - Computer simulations were conducted based on least square method to evaluate the position detection accuracy.
  - Positioning accuracy depends on the number of UAVs, location of UAVs, and other setting parameters for UAVs.
- **Positioning accuracy index (PAI)**
  - For the purpose of investigating an appropriate arrangement of UAVs, which can be used as an index for position detection accuracy instead of DOP for GPS.
- **Maximum positioning error estimation method** based on the **PAI**
  - To estimate the statistical and quantitative performance of the positioning error distance.
  - Computer simulations were conducted assuming various types of flying UAVs, such as a circularly orbiting model and a figure-eight route model, to confirm the adequate flight model and the number of UAVs by using maximum positioning error estimation method

Purpose of study

- Positioning accuracy could be improved by measuring multiple Doppler shifts at different local times, which is equivalent to increasing the number of UAVs.
- Computer simulations are conducted using the least-squares method to evaluate positioning accuracy improvement based on multiple observations of Doppler shift
  - Assuming two UAVs flying with a figure-eight route model at different initial positions
  - Estimate the position detection accuracy by the maximum positioning error estimation method for fundamental analysis
- From the simulation results, it would be confirmed that the Doppler-shift-based position detection method with the multiple-observation procedure presents much better performance in comparison with the conventional single-measurement method.

Position detection method using Doppler shifts

Conceptual model for position detection system using two UAVs

Doppler frequency value $fd_i(t_j)$ observed between UAVs and user at local time of t_j

$$fd_i(t_j) = -\frac{\left(\boldsymbol{V_i}(t_j)\cdot\tilde{\boldsymbol{u}}_{\boldsymbol{i}}(t_j)\right)}{\lambda} = -\frac{V_{xi}(t_j)(X_i(t_j)-x)+V_{yi}(t_j)(Y_i(t_j)-y)+V_{zi}(t_j)(Z_i(t_j)-z)}{\lambda\sqrt{(X_i(t_j)-x)^2+(Y_i(t_j)-y)^2+(Z_i(t_j)-z)^2}}$$

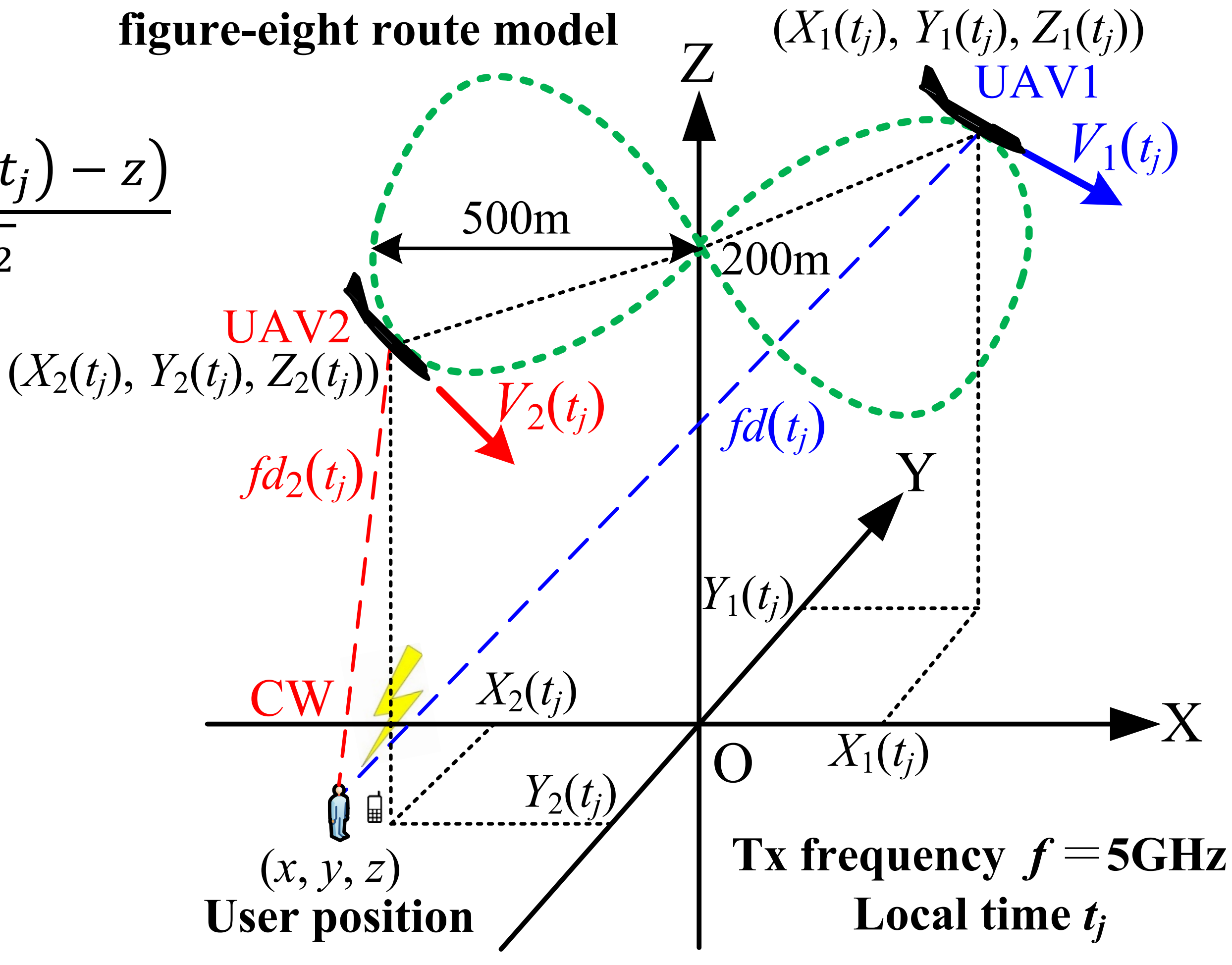
- User position : $\boldsymbol{u} = (x, y, z)$
- Location vector of the UAV at the local time of "t_j ":

$$\boldsymbol{U}_i(t_j) = \left(X_i(t_j), Y_i(t_j), Z_i(t_j)\right)$$

- Velocity vector of UAV at "t_j " :

$$\boldsymbol{V}_i(t_j) = \left(Vx_i(t_j), Vy_i(t_j), Vz_i(t_j)\right)$$

- $\tilde{\boldsymbol{u}}(t_j) = (\boldsymbol{U}(t_j)-\boldsymbol{u})/|\boldsymbol{U}(t_j)-\boldsymbol{u}|$:
  eigen vector from user position to UAV
- Transmission frequency : 5GHz



Conceptual model for a position detection system using two UAVs with a figure-eight route model

Position detection procedure based on least mean square method

- To perform the least-squares method, it is necessary to partially differentiate equation with respect to *x*, *y*, and *z*, as shown in the following:

$$\begin{cases} \dfrac{\partial f d_i(t_j)}{\partial x} = -\dfrac{-V_{xi}(t_j) r_i(t_j) + \left(X_i(t_j) - x\right) s_i(t_j)/r_i(t_j)}{\lambda\{r_i(t_j)\}^2} \\ \dfrac{\partial f d_i(t_j)}{\partial y} = -\dfrac{-V_{yi}(t_j) r_i(t_j) + \left(Y_i(t_j) - y\right) s_i(t_j)/r_i(t_j)}{\lambda\{r_i(t)\}^2} \\ \dfrac{\partial f d_i(t_j)}{\partial z} = -\dfrac{-V_{zi}(t_j) r_i(t_j) + \left(Z_i(t_j) - z\right) s_i(t_j)/r_i(t_j)}{\lambda\{r_i(t_j)\}^2} \end{cases}$$

$$s_i(t_j) = V_{xi}(t_j)\left(X_i(t_j) - x\right) + V_{yi}(t_j)\left(Y_i(t_j) - y\right) + V_{zi}(t_j)\left(Z_i(t_j) - z\right)$$

$$r_i(t_j) = \sqrt{\left(X_i(t_j) - x\right)^2 + \left(Y_i(t_j) - y\right)^2 + \left(Z_i(t_j) - z\right)^2}$$

Procedure for determining user position based on Doppler shifts

- The following equation indicates the calculated Doppler shift between the user terminal and the i^{th} UAV at a local time t_j, when the initial values of the user position for the least-squares method is set to (x^0, y^0, z^0).

$$fd_i{}^0(t_j) = -\frac{V_{xi}(t_j)\left(X_i(t_j) - x^0\right) + V_{yi}(t_j)\left(Y_i(t_j) - y^0\right) + V_{zi}(t_j)\left(Z_i(t_j) - z^0\right)}{\lambda\sqrt{\left(X_i(t_j) - x^0\right)^2 + \left(Y_i(t_j) - y^0\right)^2 + \left(Z_i(t_j) - z^0\right)^2}}$$

- Here, each measured Doppler shift between the user terminal and i^{th} UAV is assumed to be $fdm_i(t_j) = fd_i(t_j) + \Delta fdre_i(t_j)$, where $\Delta fdre_i(t_j)$ represents the variation in the Doppler shift, including that caused by the error in the location control of a UAV. Thus, the residual error, $\Delta fd_i(t_j)$, between the measured and calculated Doppler shifts can be derived as follows:

$$\Delta fd_i(t_j) = fdm_i(t_j) - fd_i{}^0(t_j)$$

Position detection procedure based on least mean square method

- To eliminate $\Delta f d_i(t_j)$, the initial value (x^0, y^0, z^0) should be changed to $(x^1, y^1, z^1) = (x^0 + \Delta x, y^0 + \Delta y, z^0 + \Delta z)$, where Δx, Δy, and Δz are the changing values for the least-squares method.
- $\Delta f d_i(t_j)$ can be expressed in terms of these variables by the following simultaneous equation:

$$\Delta f d_i(t_j) = \frac{\partial f d_i(t_j)}{\partial x} \Delta x + \frac{\partial f d_i(t_j)}{\partial y} \Delta y + \frac{\partial f d_i(t_j)}{\partial z} \Delta z$$

Position detection procedure based on least mean square method

- To solve equation, determinant matrices of G and H are expressed as the following when the number of UAVs is two ($i = 1, 2$) and when multiple observations are made for the Doppler shift at a local time $t = t_k$ and t_{k+1} ($j = k, k+1$):

$$G = \begin{bmatrix} \frac{\partial fd_1(t_k)}{\partial x} & \frac{\partial fd_1(t_k)}{\partial y} & \frac{\partial fd_1(t_k)}{\partial z} \\ \frac{\partial fd_2(t_k)}{\partial x} & \frac{\partial fd_2(t_k)}{\partial y} & \frac{\partial fd_2(t_k)}{\partial z} \\ \frac{\partial fd_1(t_{k+1})}{\partial x} & \frac{\partial fd_1(t_{k+1})}{\partial y} & \frac{\partial fd_1(t_{k+1})}{\partial z} \\ \frac{\partial fd_2(t_{k+1})}{\partial x} & \frac{\partial fd_2(t_{k+1})}{\partial y} & \frac{\partial fd_2(t_{k+1})}{\partial z} \end{bmatrix}$$

$$H = [\Delta x \ \Delta y \ \Delta z]^{\mathrm{T}}$$

Position detection procedure based on least mean square method

- From the matrices G, Δx, Δy, and Δz values for the least-squares method can be derived as:

$$[\Delta f d_1(t_k)\ \Delta f d_2(t_k)\ \Delta f d_1(t_{k+1})\ \Delta f d_2(t_{k+1})\ ]^T = F^T = GH$$

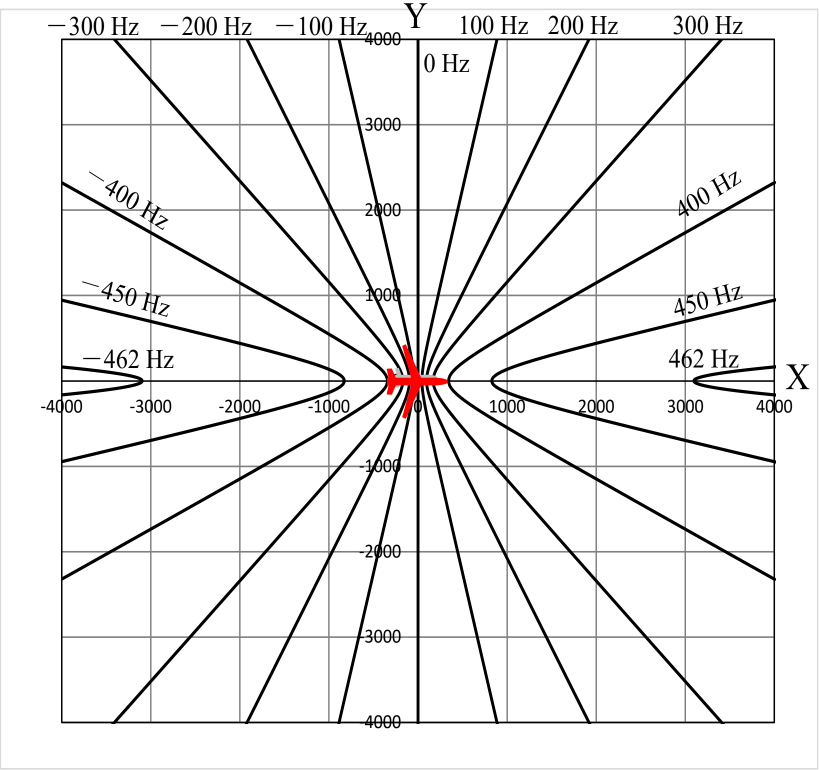
$$H = (G^T G)^{-1} G^T F^T$$

- The following operation is executed repeatedly until the values of Δx, Δy, and Δz are sufficiently close to zero:

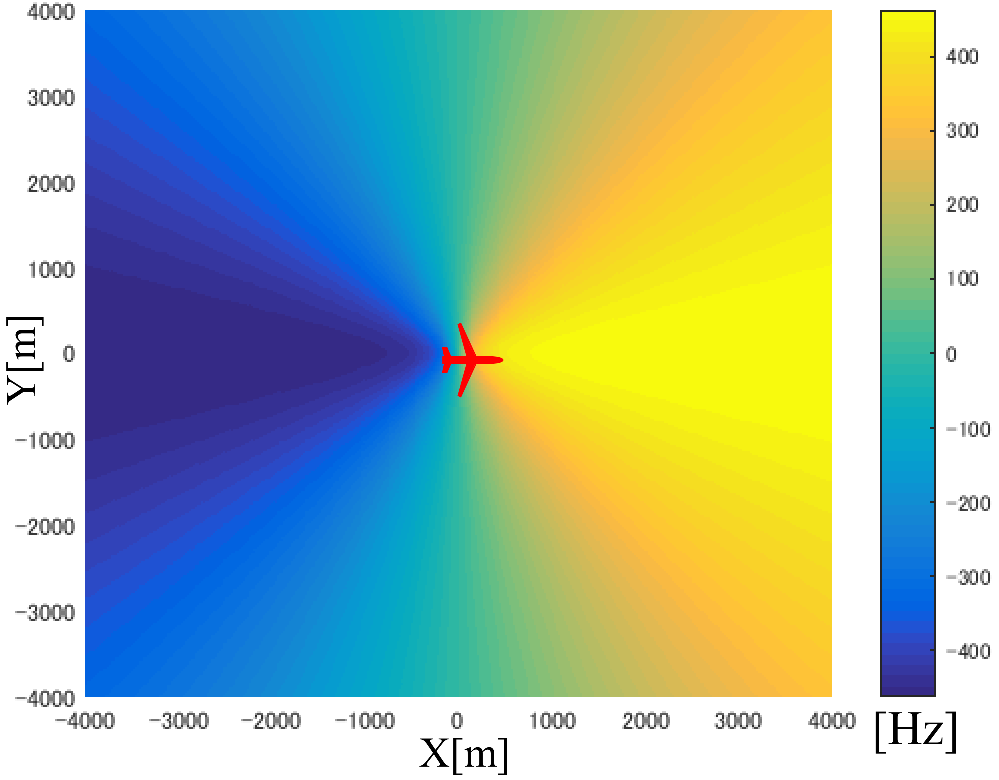
$$\begin{cases} x^1 = x^0 + \Delta x \\ y^1 = y^0 + \Delta y \\ z^1 = z^0 + \Delta z \end{cases} \Rightarrow \begin{cases} x^2 = x^1 + \Delta x \\ y^2 = y^1 + \Delta y \\ z^2 = z^1 + \Delta z \end{cases} \cdots \Rightarrow \begin{cases} x^n = x^{n-1} + \Delta x \\ y^n = y^{n-1} + \Delta y \\ z^n = z^{n-1} + \Delta z \end{cases}$$

Example of Doppler shift distribution

- Velocity of UAV V: 100 km/h (27.7m/s)
- Flight position of UAV: $(X_i, Y_i, Z_i) = (0, 0, 200)$[m]
- Direction of UAV: identical to the positive direction of the X axis
- Carrier frequency: $f_C = 5$GHz



Discrete distribution of Doppler shift on XY-plane for single UAV



Continuous distributions of Doppler shift on XY-plane

Positioning accuracy index (PAI) using gradient vector based on Doppler shift

Definition of positioning accuracy index (PAI)

- Absolute value of cosine of the inner product between two gradient vectors on the hyperboloid surface is defined as the positioning accuracy index（**PAI**） to express an influence of constellation of UAV at local time t_j for the i^{th} UAV is expressed by the following:

- A gradient vector $\nabla f d_i(t_j)$ of the hyperboloid surface, each of which is formed based on a measured Doppler shift, at an arbitrary point (x, y, z) in the evaluation target area is expressed as

$$\nabla f d_i(t_j) = \frac{\partial f d(t_j)}{\partial x}\boldsymbol{i} + \frac{\partial f d(t_j)}{\partial y}\boldsymbol{j} + \frac{\partial f d(t_j)}{\partial z}\boldsymbol{k}$$

Tangent vector

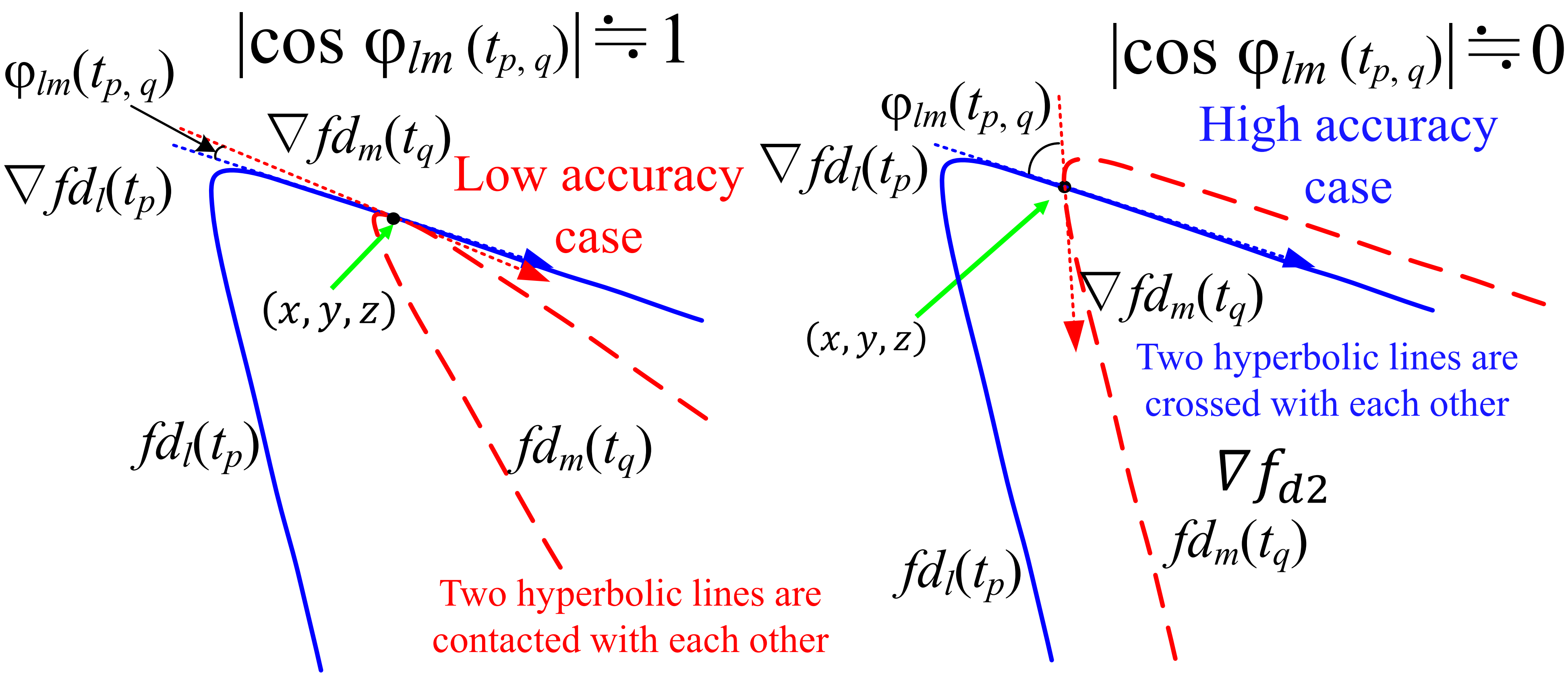
- Absolute value of cosine of the inner product between two gradient vectors formed by Doppler shifts for the l^{th} and m^{th} UAV at a local time $t = t_p$ and $t = t_q$ for (x, y, z) is defined as:

$$\left|\cos\varphi_{lm}(t_{p,q})\right| = \frac{\left|\nabla f d_l(t_p) \cdot \nabla f d_m(t_q)\right|}{\left|\nabla f d_l(t_p)\right|\left|\nabla f d_m(t_q)\right|}$$

PAI

$\varphi_{lm}(t_{p,q})$: An angle between two gradient vectors $\nabla f d_l(t_p)$ and $\nabla f d_m(t_q)$

Relationship between PAI and positioning error



Positioning Accuracy Index
(PAI)

$$|\cos \varphi_{lm}(t_{p,q})| = \frac{|\nabla f d_l(t_p) \cdot \nabla f d_m(t_q)|}{|\nabla f d_l(t_p)||\nabla f d_m(t_q)|}$$

$$\boldsymbol{\nabla} f d_i(t_j) = \frac{\partial f d(t_j)}{\partial x}\boldsymbol{i} + \frac{\partial f d(t_j)}{\partial y}\boldsymbol{j} + \frac{\partial f d(t_j)}{\partial z}\boldsymbol{k}$$

$\cos \theta = 0.95$
$\Rightarrow \theta = 18.2°$

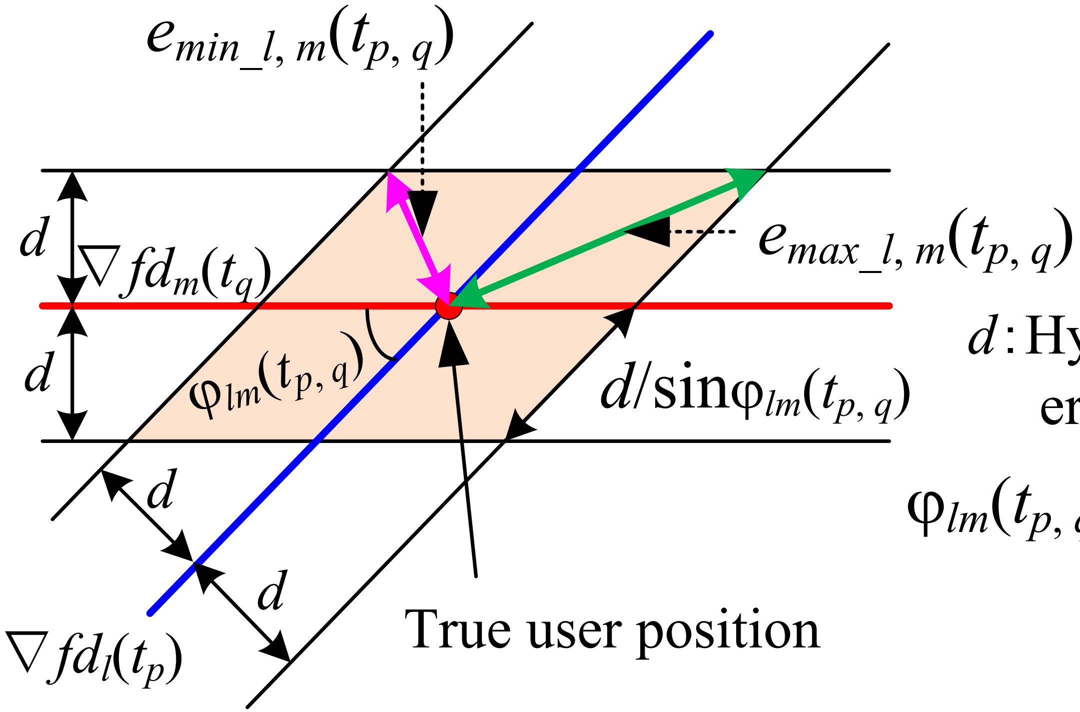
Maximum and minimum positioning error estimation method

Definition of positioning error estimation method

- Relationship between maximum and minimum estimated positioning errors and an angle of $\varphi_{lm}(t_{p,q})$ when a location error of the UAV is assumed to be a constant value of d [m]

- Maximum and minimum positioning errors and are estimated as following equations:

$$e_{\max\_l,m}(t_{p,q}) = \frac{2d\cos\left(\left|\varphi_{lm}(t_{p,q})\right|/2\right)}{\sin\left|\varphi_{lm}(t_{p,q})\right|}, \qquad e_{min\_l,m}(t_{p,q}) = \frac{2d\sin\left(\left|\varphi_{lm}(t_{p,q})\right|/2\right)}{\sin\left|\varphi_{lm}(t_{p,q})\right|}$$

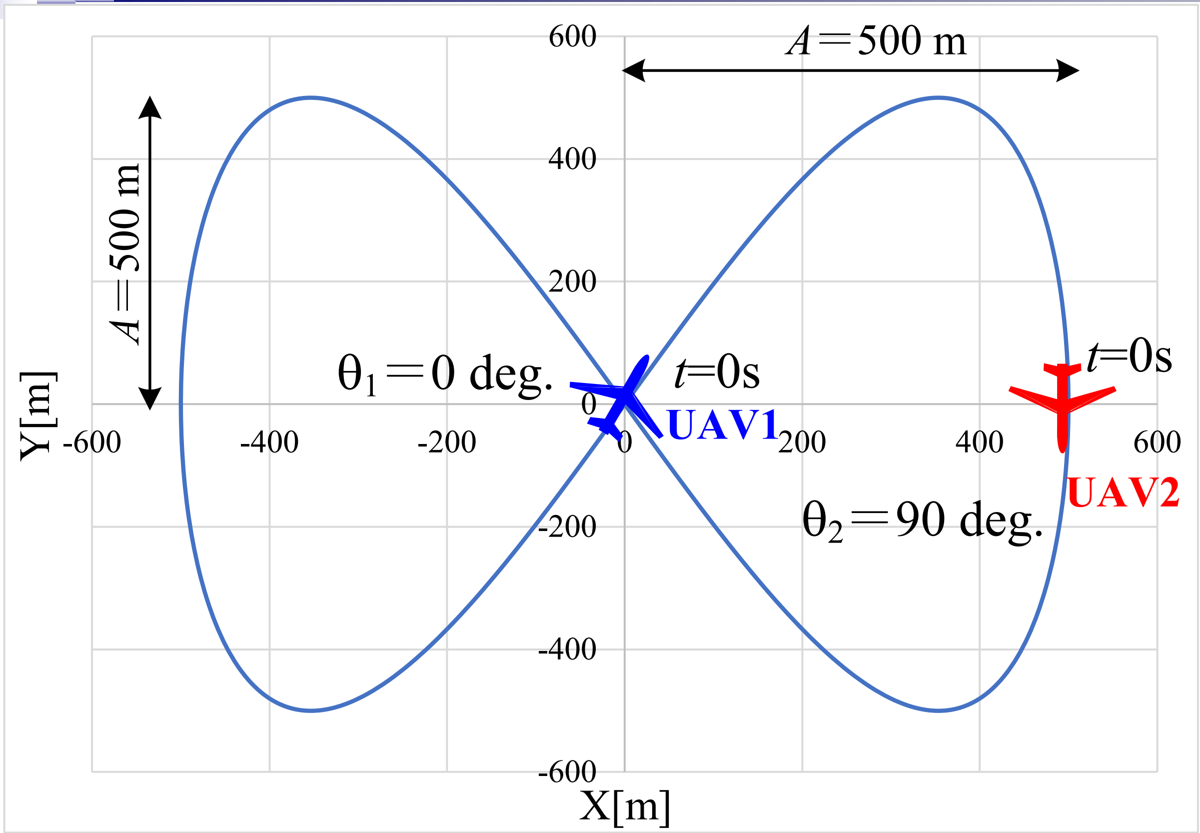


d : Hypothetical location error of UAV [m]

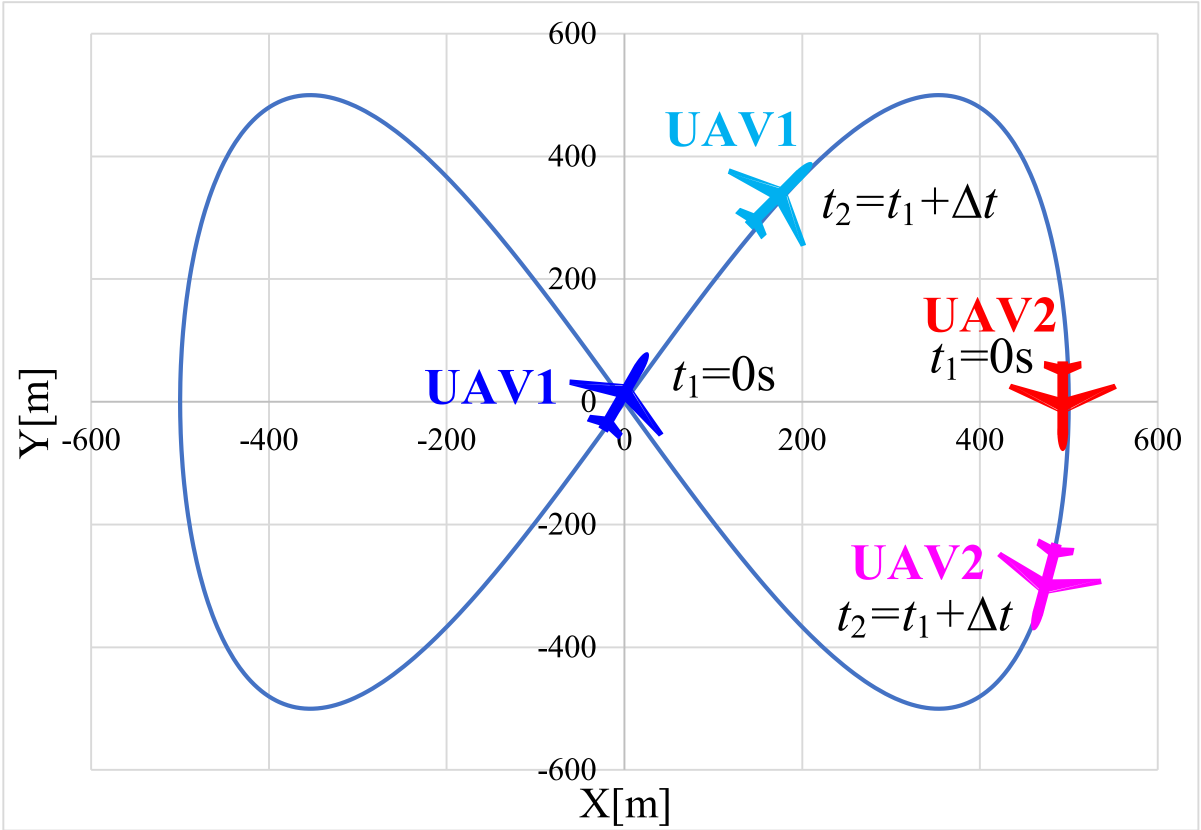
$\varphi_{lm}(t_{p,q})$: Angle between two gradient vectors

Simulation model and simulation condition

Simulation model



Initial locations of two UAVs



Relationship between multiple observations of Doppler shift and locations of two UAVs

- Velocity V=100km/h ・ Carrier frequency f_C=5GHz ・ Altitude of UAV is 200 m
- Evaluation duration for the simulation is $T = 2\pi r/v$ where the distance r = 500 m
- Evaluation target area: 64 km^2 (8 × 8 km on the XY-plane)
- Evaluation points=641,601 points (evaluation interval: 10 m for XY)

Explanations of location of UAV

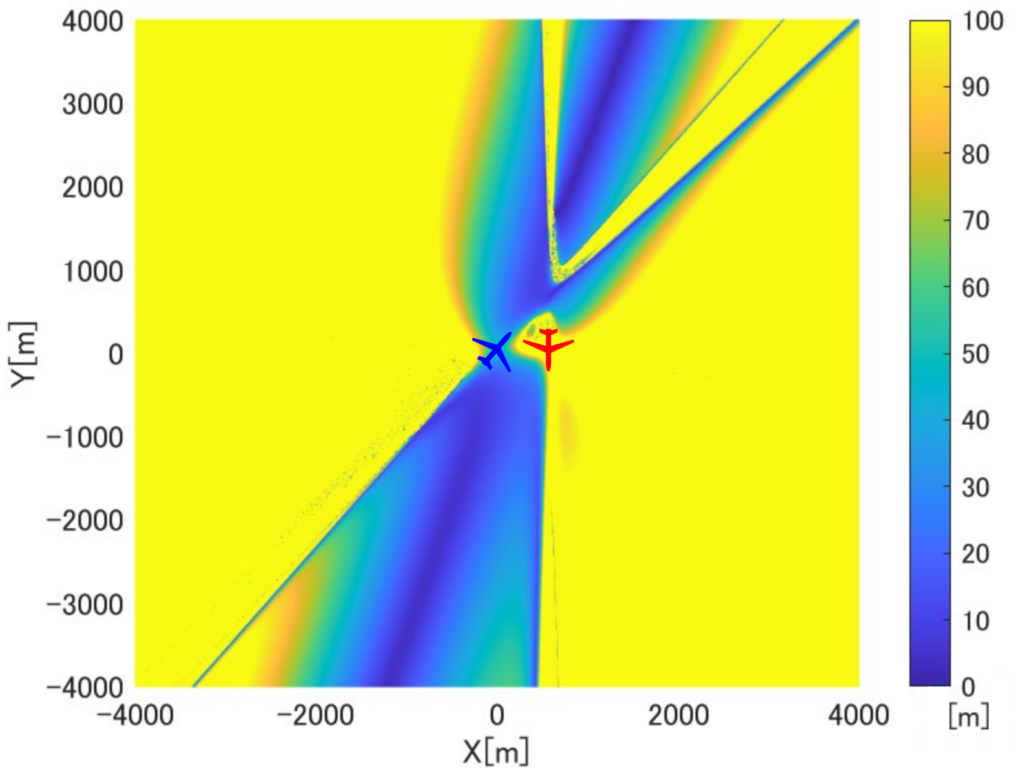
- Location vector, $\boldsymbol{U_i}(t_j) = \left(X(t_j), Y(t_j), Z(t_j)\right)$, can be expressed as:

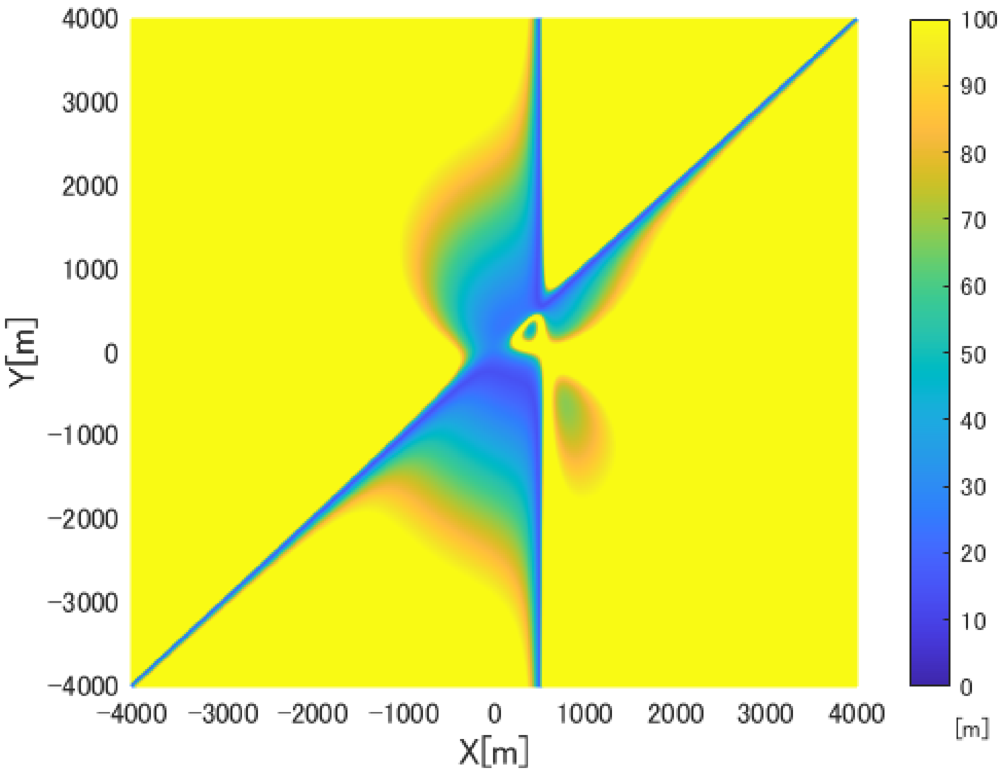
$$\begin{cases} X_i(t_j) = A\sin\{\omega_i(t_j) + \theta_i\} + X_{i0} \\ Y_i(t_j) = A\sin\{2\omega_i(t_j) + 2\theta_i\} + Y_{i0}. \\ Z_i(t_j) = Z_{i0} \end{cases}$$

- (X_{i0}, Y_{i0}, Z_{i0}) : center position, θ_i : initial phase, $\omega_i(t_j)$: angular velocity of the i^{th} UAV
- In this study, the above parameters are set to $(X_{10}, Y_{10}, Z_{10}) = (X_{20}, Y_{20}, Z_{20}) = (0,0,200)$, $\theta_1 = 0°$, $\theta_2 = 90°$, and $\omega_i(t_j) = v/A = 27.78/500 \cong 5.56 \times 10^{-2}$[rad/s]
- The observation interval of Doppler shift : $\Delta t = t_{k+1} - t_k$, where t_k and t_{k+1} indicate the successive two-measurement local times at $j = k$ and $j = k + 1$, respectively.
- Position-control error of $+0.02$ rad, which is equivalent to a location error of about 10 m for a UAV, is simply added to the phase part of the flight position of UAV1 and UAV2 to evaluate the influence of UAV position-control error that causes measurement error in the Doppler shift.

Simulation results

Positioning error distribution of conventional method when $t = 0$ s

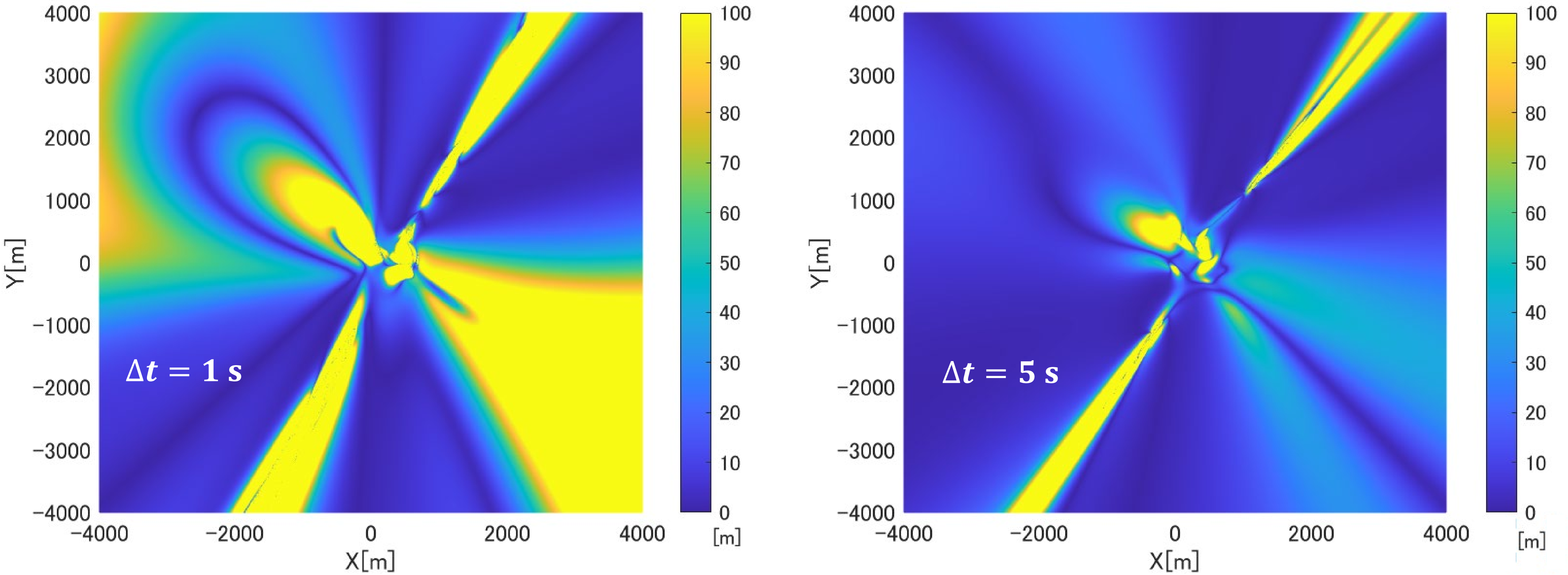




- **Blue**: high accuracy
- **Yellow**: low accuracy
- Large positioning errors occurred in almost all of the evaluation target area when the least mean square method was applied

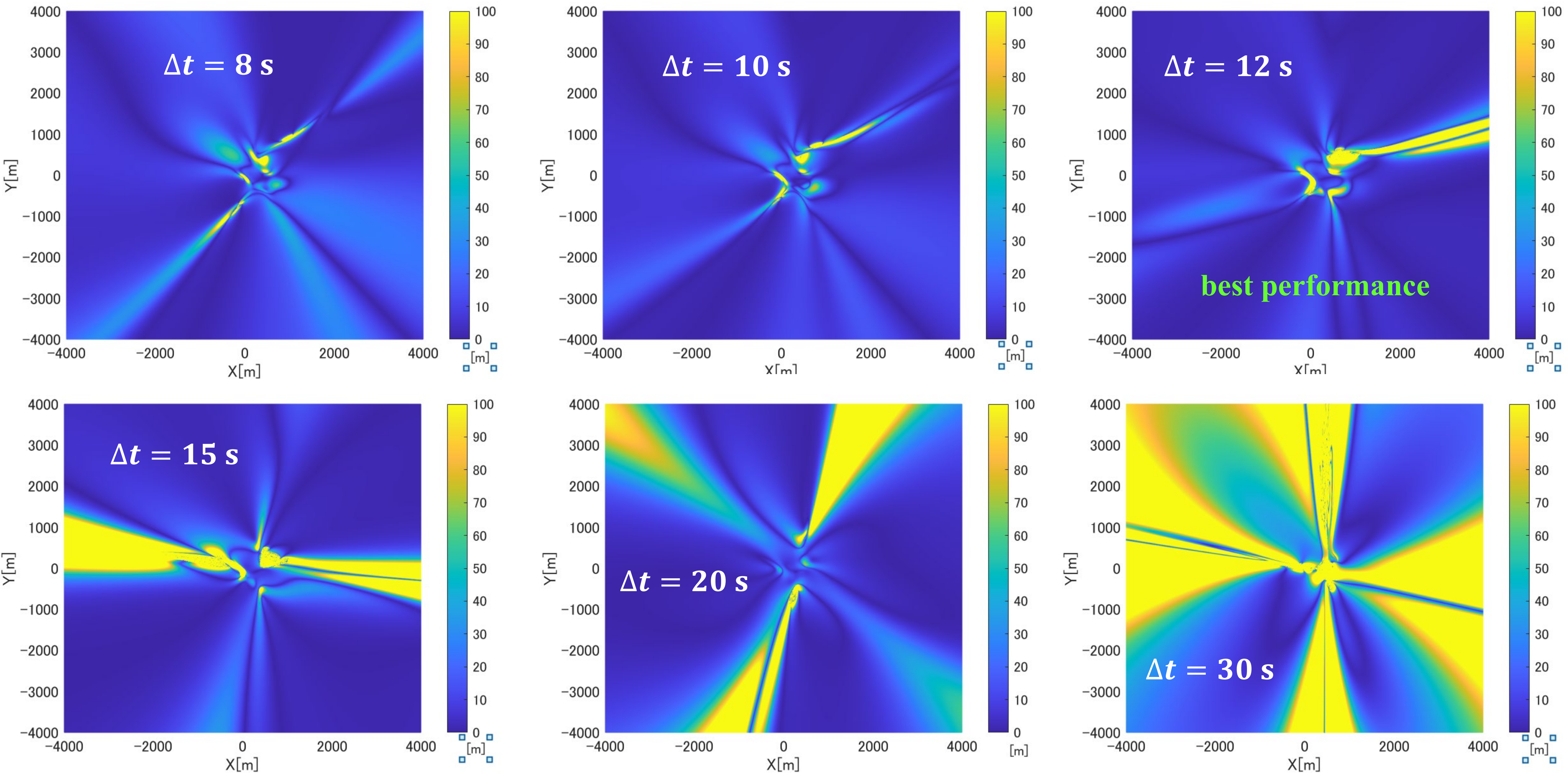
- Maximum estimated positioning error for the **conventional method** (d = 10 m)
- Maximum estimated positioning error covers the distribution features of the result of least mean square method
- High correlation exists between these performances

Error distributions of the proposed position detection procedure

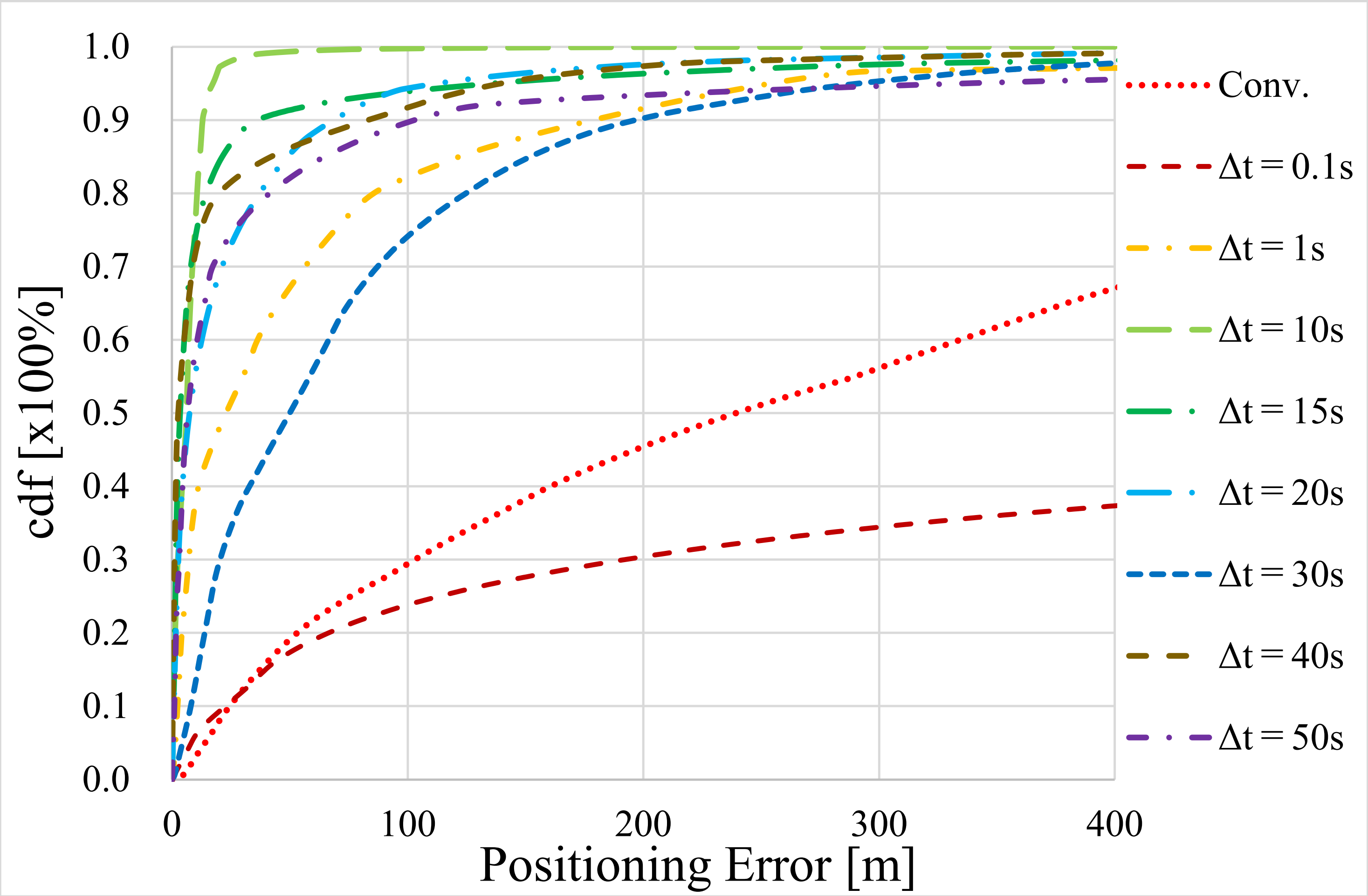


- By applying the **multiple-observation method**, the position detection accuracy is improved in accordance with the Doppler shift observation interval Δt.
- The area with large positioning error shrinks, even though the observation interval is just 1.0 s

Error distributions of the proposed position detection procedure

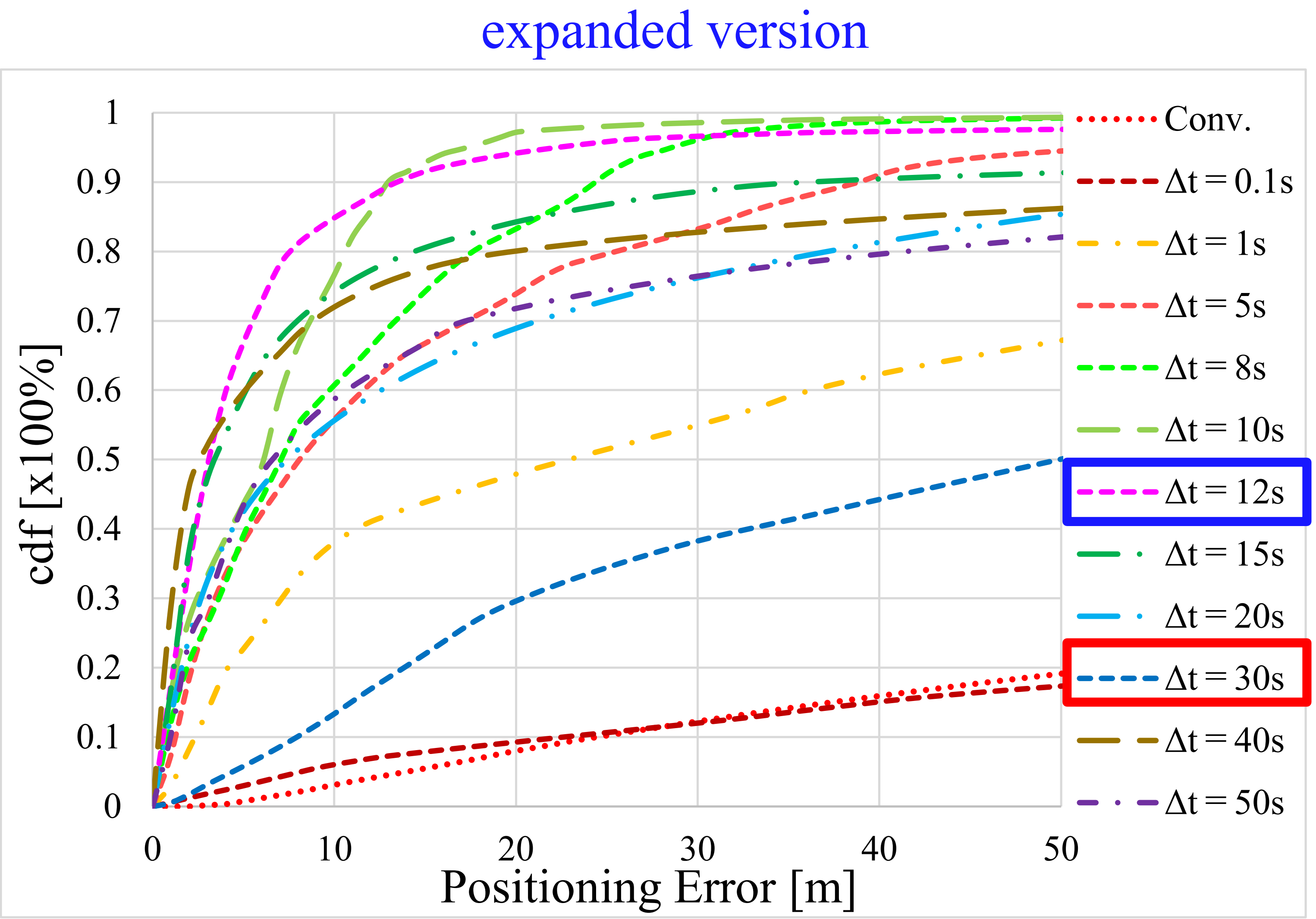


CDF performance of position detection error with different Δt



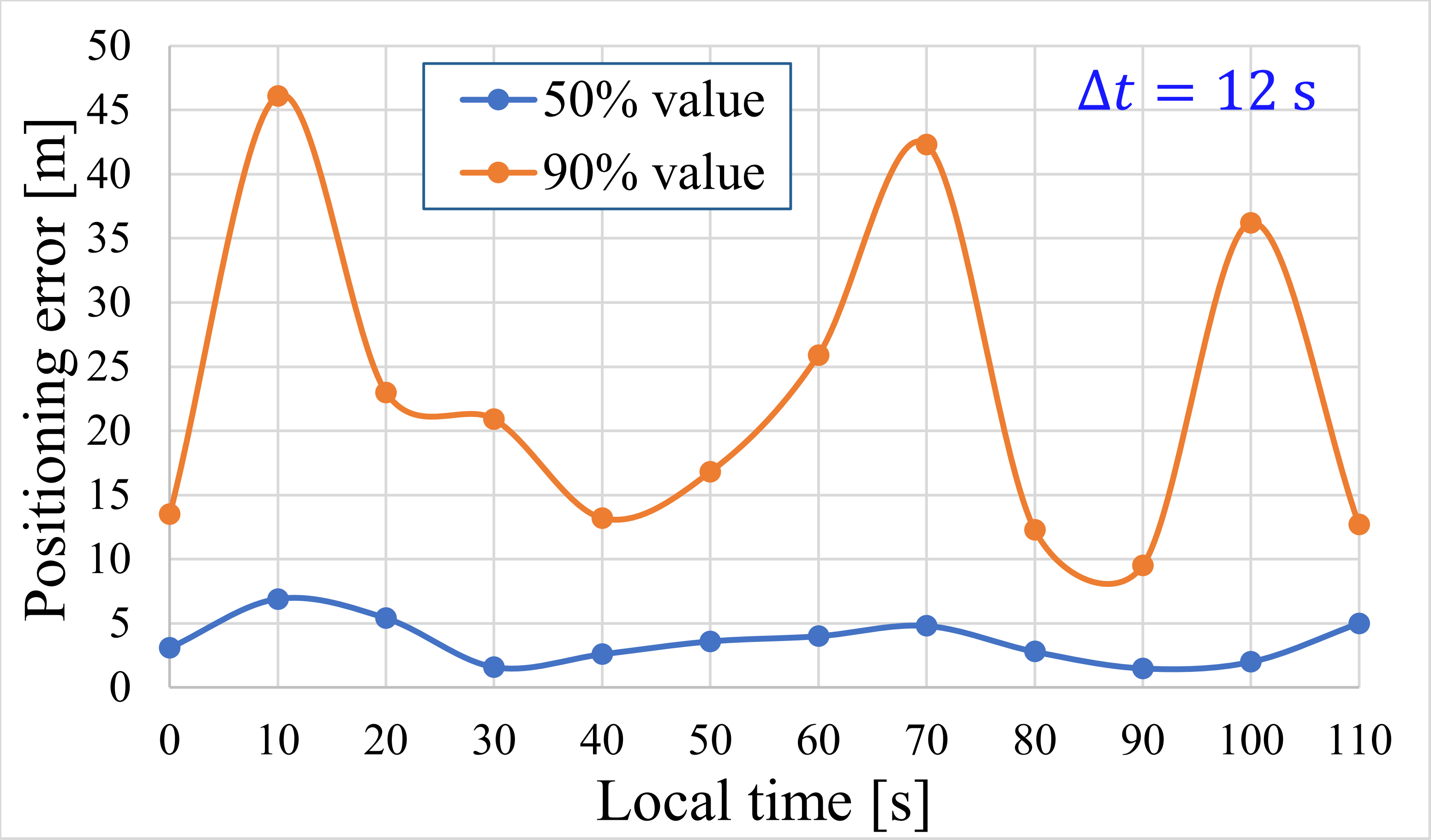
- CDF of the conventional method is worse than the multiple-observation method and exhibits almost the same performance as the case of $\Delta t = 0.1$ s.
- CDF performance of the positioning error improves in accordance with Δt because the distributions of the Doppler shift for each UAV become different when Δt becomes large.
- Locations of UAVs get close when Δt approaches approximately 28 s, which is quarter the amount of the circular duration of 113 s, and then the positioning accuracy gets worse.

CDF performance of position detection error with different Δt



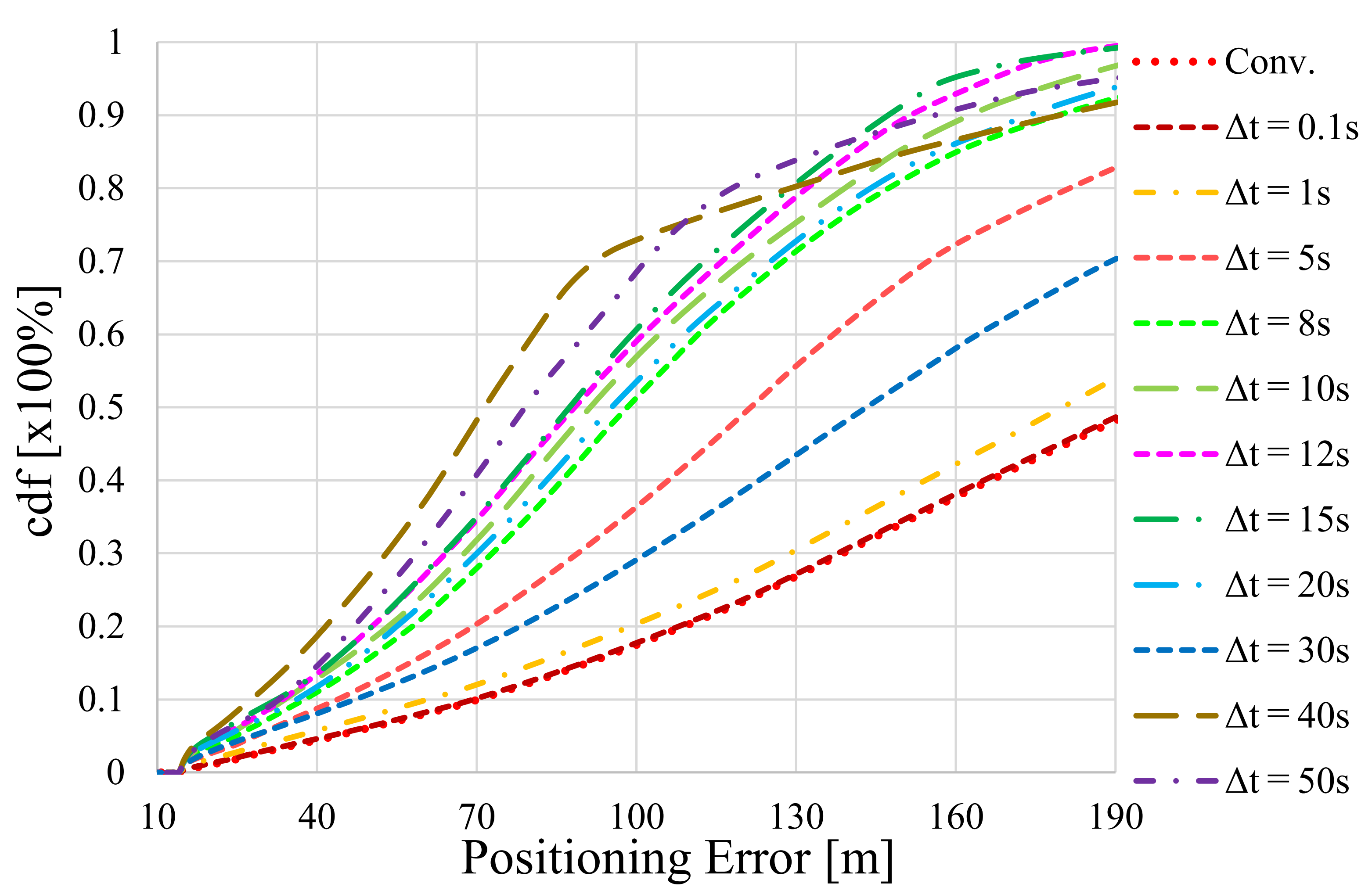
- The graph of $\Delta t = 12$ s presents the **best performance** and the graph of $\Delta t = 30$ s indicates the **worst performance** in the rage from Δt of 1 to 50 s for the multiple Doppler shift measurement method.
- Observation interval of the Doppler shift would be $\Delta t = 12$ s in the figure-eight route model and initial positions of UAV1 and UAV2 for simulation evaluation

Performance of the positioning error with CDF = 50% and 90%



- For the performance of CDF = 50%, position detection errors show **very good performance** and they change between **~2–7 m** over time.
- For the performance of CDF = 90%, the position detection error fluctuates between **~10–45 m** depending on the constellation of the UAVs at t_k and t_{k+1}.
- The worst result is much better than the case of the conventional method.

CDF performance of the maximum estimated positioning error



CDF performance of the maximum estimated positioning error with different observation intervals, Δt, when the location error of the UAV, when $d = 10$ m.

Minimum value of $\boldsymbol{e_{max\_l,m}(t_{p,q})}$ among the case of $(l, m, p, q) = \{(1,2,k,k), (1,2,k+1,k+1), (1,2,k,k+1), (1,2,k+1,k), (1,1,k,k+1), (2,2,k,k+1)\}$ is simply selected as the representative of $e_{\max\_l,m}(t_{p,q})$ for the Doppler-shift multiple-observation method.

Position detection accuracy is improved when Δt gets larger and the CDFs of **$\Delta t \approx$ 10–15 s** show almost the best performance compared to other observation intervals.

Conclusion

- Computer simulations were conducted to evaluate a positioning accuracy improvement effect based on multiple observations of Doppler shift using the least-square method.
- Estimated the position detection accuracy by the maximum positioning error estimation method for fundamental analysis.
- Doppler-shift-based position detection method with the multiple-observation procedure presents much better performance in comparison with the conventional single-measurement method.
- CDF performance of the simulation model shows good performance when the Doppler shift observation interval, Δt, is larger than 1s and shows the best performance in the case of $\Delta t = 12$ s at any local time.
- In the future, further investigations will be conducted with the aim of designing the best flight route for the proposed multiple-observation method.
- Further, environmental degrading conditions will be considered to clarify the practicality of the proposed position detection system using multiple Doppler shifts in the UAS.

Acknowledgement

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THANK YOU FOR YOUR KIND ATTENTION